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Fluid-Structure Interaction in Computational Acoustics Including Viscothermal Wave Propagation Using Finite and Boundary Elements

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Abstract
In this paper we present the threefold coupling of a boundary element formulation for acoustic radiation and scattering with a standard finite element formulation for harmonic structural vibrations as proposed in [1], [2] and [3] with a finite element formulation for viscothermal wave propagation.

This threefold coupling allows to perform acoustic analyses of coupled fluid-structure interaction problems with respect to the vibrations of an elastic structure and the sound field in the surrounding medium including viscothermal effects which typically arise in thin layers and narrow tubes. In addition, the reliable implementation into the commercial software package *NAD*work/Acoustics [4] of the proposed coupled formulation for some numerical examples is shown.

1 Introduction

Viscothermal effects must not be neglected for sound wave propagation in spaces whose dimensions in at least one direction are in the same size as the thickness of the boundary layers, in which these effects typically arise. As the thickness of the boundary layers is generally very small (for air between 0.5 mm and 0.02 mm), these effects are important for sound fields in thin layers and narrow tubes.

The constitutive equations describing the sound waves with viscous and thermal losses are [5]

- the linearized Navier Stokes equation (equation of conservation of momentum);
- the equation of continuity (equation of conservation of mass);
- the equation of conservation of energy and
- the state equation for ideal gas.

The corresponding boundary conditions are

- velocity continuity in the normal and tangential directions at the boundary to the wall;
- isothermal walls are generally assumed.

Solution of the above boundary value problem is quite a complicated problem. Different approaches to simplify the problem are therefore proposed ([5]to [11]). The so called low reduced frequency model (LRFM) [5] is applicable to the cases where the following conditions are fulfilled:

- the acoustic wave length is substantially larger than the thickness of the layers or cross-section dimension of the tubes;
- the acoustic wave length is substantially larger than the thickness of the boundary layers.

For typical applications where viscous and thermal losses have to be considered, these conditions are generally fulfilled.

2 LRFM-FEM of Viscothermal Tubes

For viscothermal tubes the LRFM differential equation for the sound pressure (which is constant in each cross-section) with the assumption of rigid walls is ([5], p83)

$$\overline{\Delta}^{pd}\,\overline{p} - \frac{\omega^2\Gamma^2}{c_0^2}\,\overline{p} = 0 \tag{1}$$

$\overline{\Delta}^{pd}$ represents the Laplace operator in propagation direction (axial direction), $\overline{p}$ is the sound pressure, ω is the circle frequency and c_0 is the undisturbed sound velocity.

$$\Gamma = \sqrt{\frac{\gamma}{n(s\sigma)B(s)}} \tag{2}$$

is the so called propagation constant with $\gamma = c_p / c_v$ denoting the ratio of specific heats, $B(s)$ being a function depending on the shape of the cross-section and the shear wave number $s = l\sqrt{\rho_0\omega/\mu}$ and

$$n(s\sigma) = [1 + \frac{\gamma - 1}{\gamma} D(s\sigma)]^{-1} \tag{3}$$

being the polytropic constant. $D(s\sigma)$ is a function that is dependent on the shape and the size of the cross-section and the thermal properties of the medium ($\sigma = \sqrt{\mu c_p / \lambda}$ is the square root of the Prandtl number, therefore $s\sigma = l\sqrt{\rho_0 \omega c_p / \lambda}$ is dependent on the specific heat and thermal conductivity of the medium but not of its viscosity). The expressions of the functions $B(s)$ for the circular and the rectangular sections are given in [4]. The function $D(s\sigma)$ is obtained for two cases: $D(s\sigma) = B(s\sigma)$ for the isothermal case and $D(s\sigma) = -1$ for the adiabatic case. The parameters $\rho_0, c_{p,} c_v, \mu, \lambda, l$ describe the undisturbed density, the specific heat at constant pressure and that at constant volume, the dynamic viscosity, the thermal conductivity, the length scale of the cross-section of a tube or the half of the thickness of a layer under consideration respectively. After some algebraic operations equation (1) can be written to a form similar to the Helmholtz equation for wave problems in the frequency domain:

$$\overline{\Delta}^{pd}\,\overline{p} + \frac{\omega^2}{c_e^2}\,\overline{p} = 0; \tag{4}$$

$$c_e = c_0 \sqrt{\frac{-B(s)}{\gamma + (\gamma - 1)D(s\sigma)}} \tag{5}$$

is the complex effective sound speed. Substituting the velocity potential $\overline{\phi} = \overline{p}/(i\omega\rho_0)$ for the sound pressure $\overline{p}$ in equation (4) and applying the Galerkin method leads to

$$\int_{\overline{A}^{cd}} \int_{\overline{A}^{pd}} w(\overline{\Delta}^{pd}\overline{\phi} + k_e^2\overline{\phi})d\overline{A}^{pd} d\overline{A}^{cd} = 0 \tag{6}$$

where w is the weight function, $k_e = \omega / c_e$ is the effective wave number and $\overline{A}^{cd}$ is the cross-section of the tube which is supposed to be constant for each tube. All entries in equation (6) are independent of $\overline{A}^{cd}$, therefore the integration is performed only over $\overline{A}^{pd}$ (the axis of the tube), which will be written in the following as s. After one time of partial integration, equation (6) can be transformed to

$$\overline{A}\int_s \frac{dw}{ds}\frac{d\overline{\phi}}{ds}ds - k_e^2\overline{A}\int_s w\overline{\phi}ds = \overline{A}\left(\frac{d\phi}{dn}\right)_{\partial s} \tag{7}$$

$\overline{A}$ represents the area of the cross-section of the tube, n is the normal vector of the two end cross-sections pointing outwards the tube and ∂s denotes the two end points of the axis of the tube.

In the following the relation between the right hand side of equation (7) and the integrals of the particle velocity over the two end cross-sections of the tube (velocity fluxes at both end cross-sections) will be derived. The dimensionless particle velocity can be expressed to ([5], p26):

$$v^{pd}(s, x^{pd}, x^{cd}) = \frac{i}{k\gamma} A(s, x^{cd}) \nabla^{pd} p(x^{pd}); \tag{8}$$

$$v^{pd} = \frac{\overline{v}^{pd}}{c_0};\;\; k = \frac{\omega l}{c_0};\;\; \nabla^{pd} = l\overline{\nabla}^{pd} = l\frac{d}{ds};\;\; p = \frac{\overline{p}}{p_0} = i\omega\rho_0\frac{\overline{\phi}}{p_0}$$

ρ_0 and p_0 are the undisturbed pressure and density of the medium. By using these relations equation (8) can be written in dimensional form:

$$\overline{v}^{pd} = -A(s, x^{cd})\frac{d\overline{\phi}}{ds} \tag{9}$$

Now the integration of the particle velocity over the cross-section can be evaluated to

$$\overline{V}^{pd} = \int_{\overline{A}^{cd}} \overline{v}^{pd} d\overline{A}^{cd} = -\frac{d\overline{\phi}}{ds} \int_{\overline{A}^{cd}} A(s, x^{cd}) d\overline{A}^{cd} = -\frac{d\overline{\phi}}{ds} B(s)\overline{A} \tag{10}$$

where $B(s)$ is a function defined by ([5], p27)

$$B(s) = \frac{1}{A^{cd}} \int_{A^{cd}} A(s, x^{cd}) dA^{cd} = \frac{1}{\overline{A}^{cd}} \int_{\overline{A}^{cd}} A(s, x^{cd}) a\overline{A}^{cd}$$

Inserting equation (10) in equation (7) gives

$$\overline{A}\int_{s} \frac{dw}{ds}\frac{d\overline{\phi}}{ds} ds - k_e^2 \overline{A}\int_{s} w\phi ds = -\left(\frac{\overline{V}^n}{B(s)}\right)_{\partial s} \tag{11}$$

$\overline{V}^n$ describes the integral of the particle velocity in the outward normal direction over the two end sections of the tube ∂s. The discrete form of equation (11) can be written to

$$\mathbf{K\varphi} - k_e^2 \mathbf{M\varphi} = \mathbf{A\varphi} = \frac{-1}{B(s)} \mathbf{V}^n$$

$\mathbf{K}$, $\mathbf{M}$ and $\mathbf{A}$ are the so called stiffness, mass and coefficient matrix, $\boldsymbol{\varphi} = \lfloor \overline{\phi}_1, \ldots, \overline{\phi}_n \rfloor^T$ and $\mathbf{V}^n = \lfloor \overline{V}_n^1, \mathbf{0}, \overline{V}_n^n \rfloor^T$ are the vectors of the nodal values of the unknown functions $\overline{\phi}$ and $\overline{V}^n$.

Elimination of the internal unknowns from the above equation gives

$$\begin{Bmatrix} \overline{V}_1^n \\ \overline{V}_n^n \end{Bmatrix} = -B(s) \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{1n} \\ \tilde{a}_{n1} & \tilde{a}_{nn} \end{bmatrix} \begin{Bmatrix} \overline{\phi}_1 \\ \overline{\phi}_n \end{Bmatrix} \tag{12}$$

where $\tilde{a}_{1n} = \tilde{a}_{n1}$, because the matrix $\mathbf{A}$ is symmetric.

Now the coupling between the tubes and the acoustic BEM can be established. If the absolute value of the direction cosines between the normal to the BE-mesh and the outward normal to the end cross-section of the tube at the two intersection points are α_1 and α_n, then the integral of normal velocity at these points for the BEM equation system can be written to

$$\begin{Bmatrix} \overline{V}_1^n \\ \overline{V}_n^n \end{Bmatrix}_{BEM} = \begin{Bmatrix} \alpha_1 \overline{V}_1^n \\ \alpha_n \overline{V}_n^n \end{Bmatrix} = -B(s) \begin{bmatrix} \alpha_1 \tilde{a}_{11} & \alpha_1 \tilde{a}_{1n} \\ \alpha_n \tilde{a}_{n1} & \alpha_n \tilde{a}_{nn} \end{bmatrix} \begin{Bmatrix} \overline{\phi}_1 \\ \overline{\phi}_n \end{Bmatrix} = \begin{bmatrix} \hat{a}_{11} & \hat{a}_{1n} \\ \hat{a}_{n1} & \hat{a}_{nn} \end{bmatrix} \begin{Bmatrix} \overline{\phi}_1 \\ \overline{\phi}_n \end{Bmatrix} \tag{13}$$

where $\hat{a}_{1n} = \hat{a}_{n1}$, when $\alpha_1 = \alpha_2$. According to [2] the right hand side vector of the BEM equation system can be written to (the contribution of incident waves is ignored, as it is here not relevant)

$$f_1 = \int_{S_1} \vartheta(x) \left\{ \tau\alpha(x) v_0(x) + \int_{S_1} \left[\tau G(y,x) A(x) + H^T(y,x) \right] v_0(y) dS_y \right\} dS_x;$$

$$f_2 = \int_{S_2} \vartheta(x) \left\{ v_0(x) + \tau \int_{S_1} \left[G(y,x) \tilde{A}(x) + H^T(y,x) \right] v_0(y) dS_y \right\} dS_x;$$

$$f_3 = \int_{S_2} \vartheta(x) \tau \int_{S_1} G(y,x) \overline{A}(x) v_0(y) dS_y dS_x \tag{14}$$

At nodal points connected by tubes the integral of the normal velocity can be expressed to

$$\int_{S_1(S_2)} v_0 dS_{x(y)} = (\overline{V}^n_{1(n)})_{BEM} = \lfloor \hat{a}_{1(n)1} \quad \hat{a}_{1(n)n} \rfloor \begin{Bmatrix} \overline{\phi}_1 \\ \overline{\phi}_n \end{Bmatrix} \tag{15}$$

Using equation (15) in equation (14) gives

$$f_1 = \tau\alpha(x) \lfloor \hat{a}_{1(n)1} \quad \hat{a}_{1(n)n} \rfloor \begin{Bmatrix} \overline{\phi}_1 \\ \overline{\phi}_n \end{Bmatrix} + \int_{S_1} \vartheta(x)\left[\tau G(y,x)A(x) + H^T(y,x)\right] dS_x \lfloor \hat{a}_{1(n)1} \quad \hat{a}_{1(n)n} \rfloor \begin{Bmatrix} \overline{\phi}_1 \\ \overline{\phi}_n \end{Bmatrix};$$

$$f_2 = \lfloor \hat{a}_{1(n)1} \quad \hat{a}_{1(n)n} \rfloor \begin{Bmatrix} \overline{\phi}_1 \\ \overline{\phi}_n \end{Bmatrix} + \tau \int_{S_2} \vartheta(x)\left[G(y,x)\widetilde{A}(x) + H^T(y,x)\right] dS_x \lfloor \hat{a}_{1(n)1} \quad \hat{a}_{1(n)n} \rfloor \begin{Bmatrix} \overline{\phi}_1 \\ \overline{\phi}_n \end{Bmatrix};$$

$$f_3 = \tau \int_{S_2} \vartheta(x) G(y,x) \overline{A}(x) dS_x \lfloor \hat{a}_{1(n)n} \quad \hat{a}_{1(n)n} \rfloor \begin{Bmatrix} \overline{\phi}_1 \\ \overline{\phi}_n \end{Bmatrix} \tag{16}$$

The integral in equation (16) need to be carried out only when the considered nodal point is located at the surface S_1 (the surface discretized by using surface elements). When the mesh consists only of middle face elements these integrals disappear ([1], [2] and [3]). As $\overline{\phi}_1$ and $\overline{\phi}_n$ are unknown, the above terms must be removed to the left hand side of the BEM equation system.

3 LRFM-FEM of Viscothermal Layers

According to equations (2.28), (2.29), (2.2) and (2.3) of [5] the sound pressure in a layer (which is constant along the thickness of the layer) obeys the following dimensionless equation

$$\Delta^{pd} p(x^{pd}) - k^2\Gamma^2 p(x^{pd}) = ikn(s\sigma)\Gamma^2 R; \tag{17}$$

$$\Delta^{pd} = l^2\overline{\Delta}^{pd}; \quad p = \overline{p}/p_0; \; k = \omega l / c_0; \; \Gamma = \sqrt{\frac{\gamma}{n(s\sigma)B(s)}};$$

$$n(s\sigma) = \left[1 + \frac{\gamma - 1}{\gamma} D(s\sigma)\right]; \quad B(s) = \frac{\tanh(s\sqrt{i})}{s\sqrt{i}} - 1;$$

$D(s\sigma) = B(s\sigma)$ (for the isothermal case) or $D(s\sigma) = -1$ (for the adiabatic case);

$$s = l\sqrt{\frac{\rho_0 \omega}{\mu}}; \quad \sigma = \sqrt{\frac{\mu c_p}{\lambda}} \tag{18}$$

In the above two equations l represents the half thickness of the layer (h^+ or h^- , see Fig. 1).

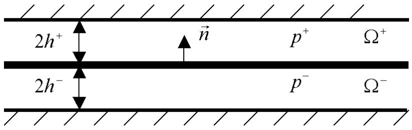

Figure 1: Layers on positive and negative side of a plate (Ω^+ and Ω^-)

By using equation (18), equation (17) can be rewritten to dimensional form:

$$\overline{\Delta}^{pd}\overline{p} + \frac{\omega^2}{c_e^2}\overline{p} = \overline{\Delta}^{pd}\overline{p} + k_e^2\overline{p} = ik\frac{\gamma p_0}{B(s)l^2}R \tag{19}$$

where $k_e = \omega / c_e$ is the effective wave number, c_e is the effective sound velocity in the layer defined in equation (5) and R is the force factor caused by the vibration of the boundary of the layer ([5], p27)

$$R = \frac{1}{A^{cd}} \int_{\partial A^{cd}} \vec{v} \cdot \vec{e}_n d\partial A^{cd} \tag{20}$$

In equation (20) A^{cd} represents the dimensionless thickness of the layer and ∂A^{cd} its boundary, $\vec{v}$ is the dimensionless velocity of the boundary and $\vec{e}_n$ is the outward normal to the boundary. For the case presented in Fig. 1 the following expressions can be derived for Ω^+, and Ω^- respectively:

$$R^+ = \frac{-\bar{v}_n}{2c_0} = \frac{i\omega\bar{u}_n}{2c_0}; \quad R^- = \frac{\bar{v}_n}{2c_0} = \frac{-i\omega\bar{u}_n}{2c_0}. \tag{21}$$

In equation (21) $\bar{v}_n$ represents the dimensional velocity of the structure in the positive normal direction and $\bar{u}_n$ is the corresponding displacement ($\bar{v}_n = -i\omega\bar{u}_n$). Using equation (21) in equation (19) (bearing in mind that $\bar{p} = i\omega\rho_0\bar{\phi}$) gives

$$\bar{\Delta}^{pd}\bar{\phi} + k_e^2\bar{\phi} = \bar{\Delta}^{pd}\bar{\phi} + \frac{\omega^2}{c_e^2}\bar{\phi} = \frac{i\delta\omega\bar{u}_n}{2lB}, \tag{22}$$

where $\delta = 1$ (for Ω^+) or $= -1$ (for Ω^-) is a switch if the considered layer is located at the positive or the negative side of the structure. Now the Galerkin method is employed to equation (22):

$$\int_{\bar{A}^{pd}} \int_{\bar{A}^{cd}} \psi \left(\bar{\Delta}^{pd}\bar{\phi} + \frac{\omega^2}{c_e^2}\bar{\phi} - \frac{i\delta\omega\bar{u}_n}{2lB} \right) d\bar{A}^{cd} d\bar{A}^{pd} = 0$$

Taking into consideration that all variables in the above equation are independent of $\bar{A}^{cd}$ and that $\int_{\bar{A}^{cd}} d\bar{A}^{cd} = 2l$, the above equation can be written to ($\bar{A}^{pd}$ is replaced by S, the middle face of the layer under consideration)

$$\int_S 2l\psi \left(\bar{\Delta}^{pd}\bar{\phi} + \frac{\omega^2}{c_e^2}\bar{\phi} - \frac{i\delta\omega\bar{u}_n}{2lB} \right) dS = 0$$

Before carrying out the partial integration it is assumed that the thickness of each layer is constant and the surface S can consist of middle faces of more than one layer. Between two neighboring layers of different thickness the condition of mass conservation (continuity of flux) at their common boundary must be fulfilled. After partial integration, the above equation is converted to

$$\int_{\partial S} 2l\psi \frac{\partial\bar{\phi}}{\partial n} d\partial S - \int_S 2l\bar{\nabla}^{pd}\psi \cdot \bar{\nabla}^{pd}\bar{\phi} dS + \omega^2 \int_S \frac{2l\psi\bar{\phi}}{c_e^2} dS - i\omega \int_S \frac{\delta\psi\bar{u}_n}{B(s)} dS = 0 \tag{23}$$

Using the same method as to derive equation (10) it can be shown that the integral of the particle velocity over the thickness (velocity flux) can be written to

$$\bar{V}^n = -\frac{\partial\bar{\phi}}{\partial n} B(s) 2l \tag{24}$$

Now it is evident that in order to guarantee the continuity of $\bar{V}^n$ at the common boundary of two layers, equation (23) must be multiplied by $B(s)$, so it is rewritten to

$$\int_S 2lB(s)\bar{\nabla}^{pd}\psi \cdot \bar{\nabla}^{pd}\bar{\phi} dS - \int_{\partial S} 2lB(s)\psi \frac{\partial\bar{\phi}}{\partial n} d\partial S - \omega^2 \int_S \frac{2lB(s)\psi\bar{\phi}}{c_e^2} ds + i\omega \int_S \delta\psi u_n ds = 0 \tag{25}$$

At the external boundary of the layers the boundary conditions must be prescribed. There are basically three categories of boundary conditions, i.e. (a) the hard boundary condition $\partial\bar{\phi}/\partial n = 0$; (b) the soft boundary condition $\bar{\phi} = 0$; (c) the admittance boundary condition $\partial\bar{\phi}/\partial n = i\omega\rho_0 a\bar{\phi}$. The boundary condition (a) and (b) can be viewed as extreme cases of the boundary condition (c): when the admittance a is equal to zero the boundary condition (c) is reduced to the boundary condition (a), when it is infinitely large the boundary condition (b) is fulfilled. After discretization of equation (25) and application of the boundary condition (c), the final algebraic equation system can be written to

$$\left(\mathbf{K}_l - i\omega\rho_0\mathbf{D}_l - \omega^2\mathbf{M}_l\right)\boldsymbol{\varphi} + i\omega\mathbf{C}_{ls}\mathbf{u} = 0\,; \tag{26}$$

$$\mathbf{K}_l = \int_S 2lB(s)\mathbf{B}_l\mathbf{B}_l^T dS\,;\;\; \mathbf{D}_l = \int_{\partial S} 2laB(s)\mathbf{n}_l\mathbf{n}_l^T d\partial S\,;$$

$$\mathbf{M}_l = \int_S \frac{2lB(s)\mathbf{n}_l\mathbf{n}_l^T}{c_e^2} dS\,;\;\; \mathbf{C}_{ls} = \int_S \delta\mathbf{n}_l\mathbf{n}_{sn}^T dS$$

$\mathbf{K}_l$, $\mathbf{D}_l$, $\mathbf{M}_l$ and $\mathbf{C}_{ls}$ are the stiffness, damping, mass and coupling matrix; $\mathbf{n}_l$ and $\mathbf{n}_{sn}$ are the vector of the shape functions of the layers and the vector of projection of shape functions of the structure in positive normal direction; $\mathbf{B}_l$ is the matrix of derivatives of the shape functions of the layers with respect to the two local coordinates in its middle face; $\boldsymbol{\varphi}$ and $\mathbf{u}$ are vectors of the nodal values of $\bar{\phi}$ and $\bar{u}$.

4 Coupling between Structure, Layer and Acoustic BEM

The FEM equation system for the structure can be written to ([1] and [2])

$$\left(\mathbf{K}_s - i\omega\mathbf{D}_s - \omega^2\mathbf{M}_s\right)\mathbf{u} = \mathbf{A}_s\mathbf{u} = \mathbf{f}_s + \mathbf{f}_p \tag{27}$$

In equation (27) $\mathbf{A}_s = \mathbf{K}_s - i\omega\mathbf{D}_s - \omega^2\mathbf{M}_s$ represents the coefficient matrix, $\mathbf{u}$ is the vector of nodal displacements, $\mathbf{f}_s$ is the nodal force vector caused by the loads other than the sound pressures and $\mathbf{f}_p$ is that caused by the sound pressure. $\mathbf{f}_p$ can be expressed to

$$\mathbf{f}_p = -i\omega\rho_0\mathbf{C}_{sl}\boldsymbol{\varphi}\text{, where the coupling matrix } \mathbf{C}_{sl} = \int_S \delta\mathbf{n}_{sn}\mathbf{n}_l^T dS$$

Therefore the coupled equation system between the layers and the structure can be written to

$$\begin{bmatrix} \mathbf{A}_s & i\omega\rho_0\mathbf{C}_{sl} \\ i\omega\mathbf{C}_{ls} & \mathbf{A}_l \end{bmatrix} \begin{Bmatrix} \mathbf{u} \\ \boldsymbol{\varphi}_l \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_s \\ \mathbf{0} \end{Bmatrix} \tag{28}$$

$\mathbf{A}_l = \mathbf{K}_l - i\omega\rho_0\mathbf{D}_l - \omega^2\mathbf{M}_l$ is the coefficient matrix of the discretization of the layers. It is evident that the coupling is symmetric, as $\mathbf{C}_{ls} = \mathbf{C}_{sl}^T$. In order to employ the method of modal superposition to equation (28), the vectors of nodal displacement of the structure and of the nodal velocity potential is expressed by their modal components:

$$\mathbf{u} = \mathbf{U}\mathbf{x}_s\,;\;\; \boldsymbol{\varphi}_l = \boldsymbol{\Phi}\mathbf{x}_l \tag{29}$$

In equation (29) $\mathbf{U}$ and $\boldsymbol{\Phi}$ are the matrices of the eigenvectors of the structure and the layers (without consideration of the viscothermal effect). Using equation (29) in equation (28) leads to the coupled equation system in the modal space:

$$\begin{bmatrix} \hat{\mathbf{A}}_s & i\omega\rho_0\hat{\mathbf{C}}_{sl} \\ i\omega\hat{\mathbf{C}}_{ls} & \hat{\mathbf{A}}_l \end{bmatrix} \begin{Bmatrix} \mathbf{x}_s \\ \mathbf{x}_l \end{Bmatrix} = \begin{Bmatrix} \hat{\mathbf{f}}_s \\ \mathbf{0} \end{Bmatrix}; \tag{30}$$

$$\hat{\mathbf{A}}_s = \mathbf{U}^T\mathbf{A}_s\mathbf{U};\ \hat{\mathbf{A}}_l = \mathbf{\Phi}^T\mathbf{A}_l\mathbf{\Phi};\ \hat{\mathbf{C}}_{sl} = \mathbf{U}^T\mathbf{C}_{sl}\mathbf{\Phi};\ \hat{\mathbf{C}}_{ls} = \mathbf{\Phi}^T\mathbf{C}_{ls}\mathbf{U};\ \hat{\mathbf{f}}_s = \mathbf{U}^T\mathbf{f}_s$$

Elimination of $\mathbf{x}_l$ from equation (30) leads to

$$\mathbf{x}_l = -i\omega\hat{\mathbf{A}}_l^{-1}\hat{\mathbf{C}}_{ls}\mathbf{x}_s;\ \left(\hat{\mathbf{A}}_s + \omega^2\rho_0\hat{\mathbf{C}}_{sl}\hat{\mathbf{A}}_l^{-1}\hat{\mathbf{C}}_{ls}\right)\mathbf{x}_s = \hat{\mathbf{A}}_s^*\mathbf{x}_s = \hat{\mathbf{f}}_s \tag{31}$$

In the following the coupling between tube and layer will be considered. Equation (25) can be written to the discretized form

$$\mathbf{A}_l\boldsymbol{\varphi} + i\omega\int_S \mathbf{n}_l\delta u_n dS = 0$$

$\int_S \delta u_n dS$ represents the displacement flux into the layer caused by the displacement of the structure. At a common nodal point of layer and tube the value of the shape function is always equal to 1 and the switch δ is not relevant (= 1), therefore, if the absolute value of the direction cosine between the normal vectors $\vec{n}_t$ and $\vec{n}_l$ is denoted by $\beta = abs(\vec{n}_t \cdot \vec{n}_l)$, then at this nodal point the above mentioned integral can be related to the velocity flux out of the tube (see Fig. 2 and equation (12)):

$$\int_S u_n ds = \frac{\overline{V}_{1(n)}^n}{-i\omega}\beta = \frac{\beta B(s)}{i\omega}\left\lfloor \tilde{a}_{1(n)1} \quad \tilde{a}_{1(n)n} \right\rfloor \begin{Bmatrix} \overline{\phi}_1 \\ \overline{\phi}_n \end{Bmatrix}$$

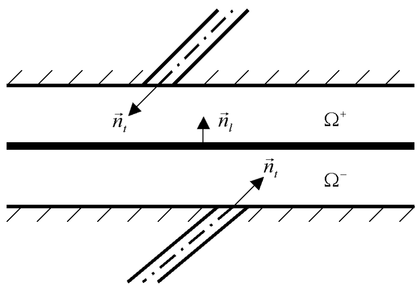

Figure 2: Coupling of layers and tubes

Combination of the above two equations leads to

$$\mathbf{A}_l\boldsymbol{\varphi} + \beta B(s)\left\lfloor \tilde{a}_{1(n)1} \quad \tilde{a}_{1(n)n} \right\rfloor \begin{Bmatrix} \overline{\phi}_1 \\ \overline{\phi}_n \end{Bmatrix} = \mathbf{A}_l\boldsymbol{\varphi} + \left\lfloor \hat{b}_{1(n)1} \quad \hat{b}_{1(n)n} \right\rfloor \begin{Bmatrix} \overline{\phi}_1 \\ \overline{\phi}_n \end{Bmatrix} = 0 \tag{32}$$

If the mesh of acoustic BEM and that of the viscothermal layers is connected by tubes, the coupling equation can be written to (see equation (16) and equation (32))

$$\begin{bmatrix} \mathbf{A}_l & \mathbf{C}_{lb} \\ \mathbf{C}_{bl} & \mathbf{A}_b \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varphi}_l \\ \boldsymbol{\varphi}_b \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{f}_b \end{Bmatrix}. \tag{33}$$

In equation (33) $\hat{b}_{11}$ (or $\hat{b}_{nn}$) is added to the corresponding diagonal term of the coefficient matrix of the layer $\mathbf{A}_l$, $\hat{b}_{1n}$ (or $\hat{b}_{n1}$) is stored in the corresponding component of the coupling matrix $\mathbf{C}_{lb}$. For $\hat{a}_{11}$ (or $\hat{a}_{nn}$), $\hat{a}_{1n}$ (or $\hat{a}_{n1}$) and the matrices $\mathbf{A}_b$ and $\mathbf{C}_{bl}$ the same principle is applicable. The number of nonzero entries in the coupling matrix $\mathbf{C}_{lb}$ or $\mathbf{C}_{bl}$ is equal to the number of tubes connecting the boundary elements and the layers. The coupling is conditionally symmetric. Application of the modal superposition to the layers leads to the following coupling equation system:

$$\begin{bmatrix} \hat{\mathbf{A}}_l & \hat{\mathbf{C}}_{lb} \\ \hat{\mathbf{C}}_{bl} & \mathbf{A}_b \end{bmatrix} \begin{Bmatrix} \mathbf{x}_l \\ \boldsymbol{\varphi}_b \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{f}_b \end{Bmatrix}; \tag{34}$$

$$\hat{\mathbf{A}}_l = \boldsymbol{\Phi}^T \mathbf{A}_t \boldsymbol{\Phi};\ \hat{\mathbf{C}}_{lb} = \boldsymbol{\Phi}^T \mathbf{C}_{lb};\ \hat{\mathbf{C}}_{bl} = \mathbf{C}_{bl}\boldsymbol{\Phi}$$

Eliminating the vector of the modal components of the layer from equation (34) gives

$$\mathbf{x}_l = -\hat{\mathbf{A}}_l^{-1}\hat{\mathbf{C}}_{lb}\boldsymbol{\varphi}_b;\ \left(\mathbf{A}_b - \hat{\mathbf{C}}_{bl}\hat{\mathbf{A}}_l^{-1}\hat{\mathbf{C}}_{lb}\right)\boldsymbol{\varphi}_b = \mathbf{A}_b^*\boldsymbol{\varphi}_b = \mathbf{f}_b \tag{35}$$

In the following the coupling between the acoustic BEM, the structural FEM and the viscothermal layers will be put together (see equation (30) and (34) and [5]). Here the coupling between BEM and layer is always realized by means of tubes. The global coupling equation in the modal spaces of the structural FEM and the FEM for the viscothermal layers can be written to

$$\begin{bmatrix} \mathbf{A}_b & i\omega\hat{\mathbf{C}}_{bs} & \hat{\mathbf{C}}_{bl} \\ i\omega\rho_0\hat{\mathbf{C}}_{sb} & \hat{\mathbf{A}}_s & i\omega\rho_0\hat{\mathbf{C}}_{sl} \\ \hat{\mathbf{C}}_{lb} & i\omega\hat{\mathbf{C}}_{ls} & \hat{\mathbf{A}}_l \end{bmatrix} = \begin{Bmatrix} \boldsymbol{\varphi}_b \\ \mathbf{x}_s \\ \mathbf{x}_l \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_b \\ \hat{\mathbf{f}}_s \\ \mathbf{0} \end{Bmatrix} \tag{36}$$

In equation (36) the coupling matrices between acoustic BEM and structural FEM and between layers and structural FEM are always symmetric. The coupling matrices between BEM and layers are however only conditionally symmetric. However, for the case that the tubes that connect the layers and BE-faces are normal to the layers, these matrices are also symmetric. The elimination of $\{x_l\}$ from equation (36) results in

$$\mathbf{x}_l = -\hat{\mathbf{A}}_l^{-1}\left(\hat{\mathbf{C}}_{lb}\boldsymbol{\varphi}_b + i\omega\hat{\mathbf{C}}_{ls}\mathbf{x}_s\right);$$

$$\begin{bmatrix} \mathbf{A}_b - \hat{\mathbf{C}}_{bl}\hat{\mathbf{A}}_l^{-1}\hat{\mathbf{C}}_{lb} & i\omega(\hat{\mathbf{C}}_{bs} - \hat{\mathbf{C}}_{bl}\hat{\mathbf{A}}_l^{-1}\hat{\mathbf{C}}_{ls}) \\ i\omega\rho_0(\hat{\mathbf{C}}_{sb} - \hat{\mathbf{C}}_{sl}\hat{\mathbf{A}}_l^{-1}\hat{\mathbf{C}}_{lb}) & \hat{\mathbf{A}}_s + \omega^2\rho_0\hat{\mathbf{C}}_{sl}\hat{\mathbf{A}}_l^{-1}\hat{\mathbf{C}}_{ls} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varphi}_b \\ \mathbf{x}_s \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_b \\ \hat{\mathbf{f}}_s \end{Bmatrix} \tag{37}$$

Equation (37)b can be abbreviated to

$$\begin{bmatrix} \mathbf{A}_b^* & i\omega\hat{\mathbf{C}}_{bs}^* \\ i\omega\rho_0\hat{\mathbf{C}}_{sb}^* & \hat{\mathbf{A}}_s^* \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varphi}_b \\ \mathbf{x}_s \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_b \\ \hat{\mathbf{f}}_s \end{Bmatrix} \tag{38}$$

Eliminating $\mathbf{x}_s$ from equation (38) finally gives

$$\mathbf{x}_s = (\hat{\mathbf{A}}_s^*)^{-1}(\hat{\mathbf{f}}_s - i\omega\rho_0\hat{\mathbf{C}}_{sb}^*\boldsymbol{\varphi}_b);\ [\mathbf{A}_b^* + \omega^2\rho_0\hat{\mathbf{C}}_{bs}^*(\hat{\mathbf{A}}_s^*)^{-1}\hat{\mathbf{C}}_{sb}^*]\boldsymbol{\varphi}_b = \mathbf{f}_b - i\omega\hat{\mathbf{C}}_{bs}^*(\hat{\mathbf{A}}_s^*)^{-1}\hat{\mathbf{f}}_s \tag{39}$$

5 Examples

The formulations of the LRFM-FEM for viscothermal tubes and layers presented in the last three sections have been implemented into the commercial software package *NAD*work/Acoustics [4]. Three examples will be presented here to show the accuracy of the numerical method.

The first example concerns two volumes connected by means of five tubes. The upper volume has a diameter of 30 mm and a height of 28.29 mm (2e4 mm^3), the corresponding parameters of the lower volume are 30 mm and 11.32 mm (8e3 mm^3). The five tubes (length = 50 mm) are of circular (diameter = 3mm) or of rectangular (4*2 mm) cross-section. At the top of the upper volume a circular plate is embedded which is vibrating as a rigid body with elastic support in the axial direction of the volume with an exciting force of 1 N. The parameters of the vibrating plate are: diameter = 20 mm; mass = 8e-5 kg; stiffness of support = 0.2 N/mm; modal damping 2%. The numerical results for the velocity of the plate in the frequency domain of 10 to 1000 Hz are shown in Fig 3.

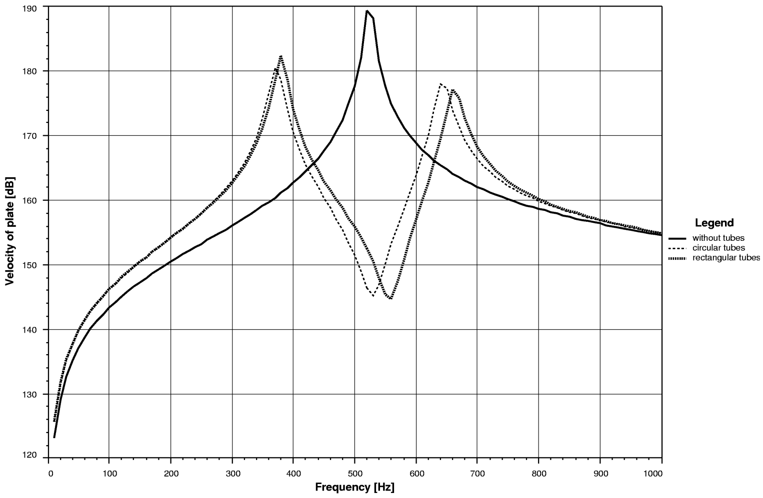

Figure 3: Two volumes connected by means of five tubes

The second example is chosen from [8]. A steel plate (464*63.5*1.6 mm) with two clamped shorter sides and two free longer sides is excited by a line load of 1 N acting at a line 75 mm away from one clamped end of the plate. The plate is backed by a thin air gap, the boundary of the air layer is acoustically soft (sound pressure = 0). The layer acts here as an attenuator due to the energy loss associated with the movement of air particles in the plane of the layer. The results for the displacement of the node lying in the middle of the load line is illustrated in Fig. 4. Compared to the results of [8], which are obtained by using the transfer matrix method , the agreement is fairly good (Fig. 5 in [8]).

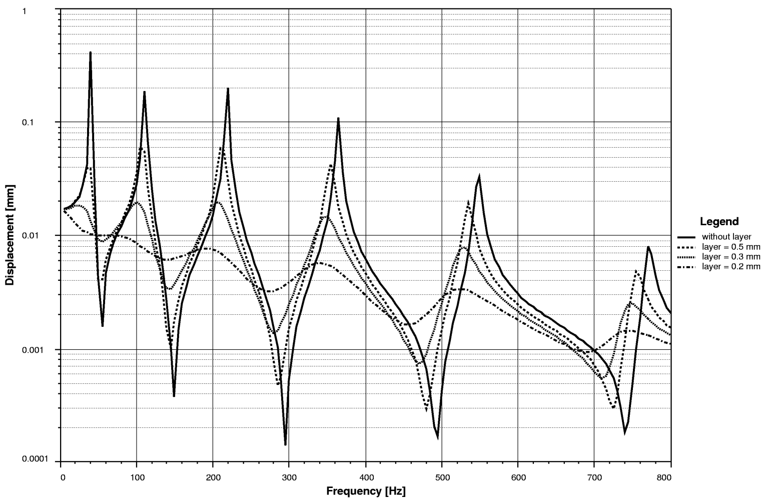

Figure 4: Steel plate with air gap

The third example corresponds to the prototype system used in [9]. It concerns the same plate as in the second example, but now the air gap is not located under the whole plate but only under a stretch of length 76 mm in the middle of the plate. The numerical results shown in the Fig 5 are in good agreement with the theoretical solution of [9], which is obtained by using the transfer matrix method (Fig. 10 in [9]).

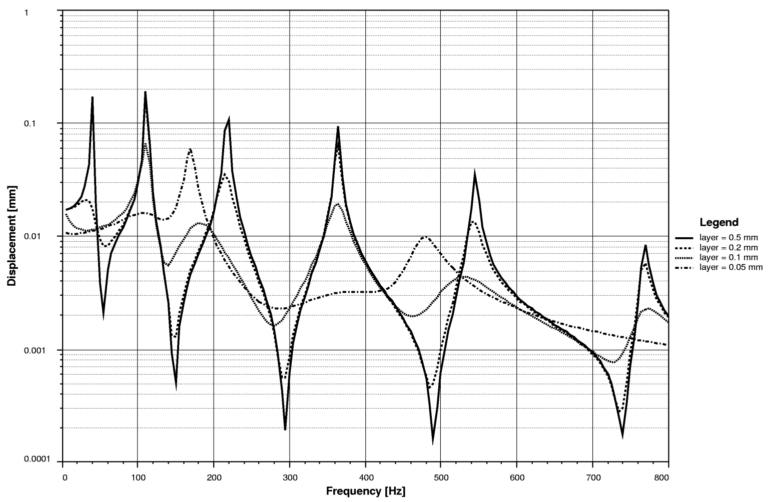

Figure 5: Steel plate with partial air gap

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