ACOUSTIC MATRIX METHODS FOR WOOFERS, TWEETERS, HORNS AND SMALL TRANSDUCERS

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ABSTRACT

This paper deals with the use of matrix methods, especially finite and boundary element methods, for the simulation of sound fields of diverse audio systems like woofers, tweeters, horns or small transducers as used in cell phones. First we will give a short overview about the theory of the numerical methods used for such predictive engineering tasks. However, the main focus is to show typical applications from industry of such methods for audio engineers. Finally we will make some remarks on the business benefits of these methods but we will also address uncertainties in our simulation theory which are in principle inherent to every simulation model.

INTRODUCTION

In the majority of cases the focus of the development work on electroacoustic transducers (loudspeakers) is influenced by the viewpoint of an electrical engineer. This seems to be due to the fact, that every loudspeaker is driven by some kind of electrical device. But a loudspeaker consists not only of electrical parts. It is build up of an electrical motor system attached to a mechanical vibration system which typically converts an electrical signal to a mechanical force which in turn excites the mechanical vibration system consisting of at least a diaphragm with some kind of support. The diaphragm itself excites the surrounding air to radiate sound. That is in very simple words the functional principle of a loudspeaker.

If we take a closer look we figure out that the following physical domains are involved: electrical, mechanical and acoustical domain. Thus this is often referred to as multiphysics. Quite interesting is the fact, that only one domain is a typical electrical domain. A recent panel discussion [1] gave a good and more general overview about the involved physical domains from a stimulus to the human ear.

In the last decades there has been spent a lot of efforts on improving the behavior of loudspeakers by means of numerical simulations. The focus was, and often enough still is, in improving the electromagnetic behavior of loudspeakers by using advanced simulation techniques. However, when doing systems optimization one must also include the treatment of the structural dynamic and sound radiation behavior by means of advanced simulation models. Thus we have to deal with a multiphysical system. So in this paper we will present a general theory based on a lumped parameter model for the electrical domain, a finite element model for the mechanical domain and a boundary element model for the acoustical domain for predictive multiphysical simulations of speakers.

THEORY

There is a long tradition to treat loudspeakers as a multiphysics device by electrical engineers. In the early seventies several technical papers were presented to the AES [2] that resulted in a model which audio engineers know today as the famous “Thiele-Small Parameters”. A.N. Thiele and R.H. Small developed parameters defining the relationship between a loudspeaker and a particular enclosure. Over time there have been a lot of developments to extend these early lumped parameter models to more sophisticated models. Especially the recent work of W. Klippel on the large signal behavior should be mentioned here.

However, all these models have one significant drawback, they are based on lumped parameters which use only a few parameters to describe a physical domain. Whereas the usage of a lumped parameter model for voice coil electromagnetics turned out to be very useful, for the mechanical domain and for the acoustical domain one can see a lot of limitations. E.g. the mechanical system is described in a lumped parameter model by scalar values of stiffness and mass. This leads to the fact, that this model is only capable of describing the piston mode of a vibration system. This will yield to significant limitations in the frequency
range where the lumped parameter model is applicable.

This fact has led to the development of distributed models based on matrix methods, where the domains are modeled via systems consisting of many degrees of freedom. We will now present a fully coupled electrical-mechanical-acoustical simulation model for loudspeakers, which uses a lumped parameter model for voice coil electromagnetics, a finite element model for the mechanical domain and a boundary element model for the acoustic domain.

In our lumped parameter model for voice coil electromagnetics the electrical force $f_e$ acting on the coil under the assumption of a constant voltage source $U$ is defined as:

$$ f_e = U \frac{Bl}{R + j\omega L} - \frac{(Bl)^2}{R + j\omega L} v = f_{e,L} - f_{e,EMF} $$

Whereas $B$ is the flux density, $l$ the length of the wire in $B$, $R$ the DC resistance of the coil, $L$ the inductance of the coil and $v$ is the velocity of the moving coil. $f_{e,L}$ is the Lorentz force and $f_{e,EMF}$ is the back EMF.

The governing equation for the mechanical vibrations, discretized by means of finite elements, can be written as follows:

$$(K^m + j\omega D^m - \omega^2 M^m)u^m = f^m$$

At a first glance there seems to be only a little difference in the governing equations by matrix methods and by lumped parameter models. However, the big difference is the dimension of the system. In the finite element governing equation stiffness, mass and damping are being described via matrices. $K^m$ is the stiffness matrix, $D^m$ is the damping matrix and $M^m$ is the mass matrix. Furthermore, $u^m$ is the vector of displacements and $f^m$ is the vector of mechanical forces exciting the system. Typically the dimension is of several of thousands degrees of freedom. In fact the governing equation is a system of equations describing the mechanical vibrations with respect to a detailed definition of the geometry discretized via finite elements. Thus it is possible to use these models for the whole audible frequency range which is typically from 20 Hz up to 20 kHz where a lot of non-pistonic and non-axisymmetric modes occur, which can’t be described via a lumped parameter model.

The principle of using matrices for describing a physical domain is also used for the acoustical domain where the Helmholtz equation is discretized by means of boundary elements:

$$ B^n p^n = f^n $$

Here $B^n$ is an acoustic coefficient matrix which defines the relationship between the vector of the sound pressure $p^n$ and a vector including effects of incident sound waves $f^n$. Thus it is possible to describe the sound radiation with respect to a detailed definition of the geometry including reflectional as well as diffractional effects.

If we now tie together all the governing equations of the different physical domains we get the following coupled system of equations describing the multiphysics of an electroacoustic transducer:

$$ \begin{bmatrix} K^m + j\omega(D^m + D^e) - \omega^2 M^m \\ C_{am} \end{bmatrix} \begin{bmatrix} u^m \\ B^n \end{bmatrix} = \begin{bmatrix} f_{e,L} \\ f^n \end{bmatrix} $$

$C_{am}$ and $C_{ma}$ are coupling matrices connecting the mechanical and acoustical domain. These coupling matrices arise from the assumption of continuity of the velocity components and of the pressure in the mechanical as well as the acoustical domain in the direction normal to the coupling surface. $D^e$ is an electromagnetic damping which has been derived from the back EMF.

**APPLICATIONS FROM INDUSTRY**

Now we will show some typical applications from industry of loudspeaker simulations based on the theory as outlined in the previous section. All examples presented here were carried out by means of a commercial software tool [3].

First we will start with the simulation example of a woofer attached to a closed cabinet. Figure 1 shows the simulation model and the sound pressure distribution at 11.2 kHz. At the windings of the voice coil the lumped parameter model for electromagnetics is coupled to the vibrating parts of the driver modeled with finite elements. All these vibrating parts are also coupled to the acoustical domain by means of boundary elements. The surface of the magnet system as well as the walls of the cabinet are assumed to be rigid. Thus only acoustical boundary elements are being used for those parts.
A comparison between simulation and measurement for the sound pressure level on axis can be seen in Figure 2.

The same modeling principle can also be used for the simulation of tweeters. A simulation model of a dome tweeter with an acoustic lens can be seen in Figure 3 where also the sound pressure distribution at 28 kHz is shown. Please note the resonance effects in the cavity of the magnet system which highly influence the mechanical vibrations and thus the sound radiation.

A similar modeling procedure can also be used for horns and compression drivers. However, for the simulation of compression drivers one has to include viscothermal effects in the compression chamber. More details about adding viscothermal effects in the proposed theory can be found in [4]. The previously described theory can be used for simulating the propagation of sound inside a horn and subsequently the sound radiation. Thus simulations of the frequency response and directivity of horns can be carried out. Figure 4 shows a simulation model of a horn where also the sound pressure distribution at 12 kHz is shown.

This simulation uses a very simple model for the compression driver, where a constant velocity distribution at the horn's throat is applied. However, as the purpose of this analysis is to simulate the pure acoustical behavior of the horn this assumption turned out to be very useful. Therefore, the model contains only acoustical boundary elements.

Yet another typical field of application are small transducers for cell phones. Figure 5 shows a vibration system of rectangular shape where also the distribution of the structural velocity at 4 kHz can be seen. Due to the specifically small sizes of transducers for cell phones special effects like the already mentioned viscothermal losses and especially mechanical nonlinearities have to be accounted for. Those small transducers will typically run at large excursions to radiate an appropriate sound level which will generate a lot of mechanical nonlinearities resulting in a significant
amount of distortion. So one of the design goals for such transducers is to optimize mechanical nonlinearities.

![Figure 5: Simulation model of small transducer (with kind permission of Sonion Horsens A/S)](image)

In the proposed theory it is possible to include mechanical nonlinearities. These nonlinearities are caused by the fact, that the vibration system significantly changes its shape when driven at large excursions. This will result in a nonlinear force excursion relation and obviously in a change of the stiffness depending on the excursion. Figure 6 shows the typical nonlinear behavior of a small transducer for a cell phone. The solid lines show the nonlinear relationship between the applied force and the excursion of the voice coil, whereas the dotted lines show the change of stiffness.

![Figure 6: Mechanical nonlinear behavior of small transducer (with kind permission of Sonion Horsens A/S)](image)

**CONCLUSION**

The accurate predictive sound simulation of woofers, tweeters, horns and small transducers by means of a coupled electrical-mechanical-acoustical simulation model has been shown within the context of this paper based on some typical applications from industry. The proposed model uses matrix methods for the mechanical domain (finite elements) and the acoustical domain (boundary elements) as well as a lumped parameter model for voice coil electromagnetics.

The business benefits of using such simulation methods are to be obvious. One effect is that the audio engineer will get a much better understanding of the physics and sources of inefficiencies of speakers which will directly lead to product enhancements. Yet another effect is that development time can be reduced significantly by using virtual prototypes which are typically build up much faster than prototypes in hardware.

One final aspect which has to be addressed here are uncertainties in the simulation models. Most of the uncertainties can be found in the mechanical domain. Of capital importance are the mechanical material properties of the vibration system. Especially the modulus of elasticity and the damping values should be treated with great care. Available methodologies for retrieving those material parameters from measurements should be used to optimize the accuracy of simulation models.

**REFERENCES**


**AUTHORS HISTORY**

Alfred J. Svobodnik (as@NADwork.at) holds a Ph.D. from Vienna University of Technology and is co-founder and CEO of NAD, a Wien/Austria based company which was established in 1990. Since 1994 he led numerous research projects on the numerical simulation of acoustical systems by means of finite and boundary elements for sound and noise applications. In 1997 he started the distribution of NADwork Simulation Suite, a commercial software tool for computational acoustics, to industry. Since 2002 he is responsible for international sales activities with a special focus on the audio industry.